Higher spin gravity, pre-Calabi-Yau, Formality and Convex geometry

Calabi–Yau day

Evgeny Skvortsov, UMONS

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Main Messages

- **Higher spin gravity** is one of ideas to construct/solve models of quantum gravity, which is heavily based on ∞-dim symmetry
- **Higher spin symmetry** is an application of the usual Deformation Quantization, also present in simple CFT's
- **Chiral HiSGRA** is a $4d$ model of quantum gravity that via AdS/CFT is related to $3d$ CFT's aka second order phase transitions (Ising). Equations of motion are given by (AKSZ) Poisson sigma-model that originates from a **pre-Calabi-Yau** algebra.
- A_∞-maps are given by Kontsevich-like integrals over convex polygons. The first two maps are thanks to (Shoikhet-Tsygan)-Kontsevich formality. It would be great to understand why

Outline

- a bit of physical context: higher spin gravities and where (why) to find them; **More:** Snowmass paper, ArXiv: 2205.01567
- Formal field theories and L_∞-algebras or Q-manifolds
- From L_∞ to A_∞ and back again
- Poisson sigma-model, pre-Calabi-Yau algebras
- Structure maps and convex geometry, Stokes theorem
- Relation to (Shoikhet-Tsygan)-Kontsevich Formality

Everything is based on the works with Alexey Sharapov, Richard Van Dongen and Arseny Sukhanov

Why higher spins?

Physical context

Why higher spins?

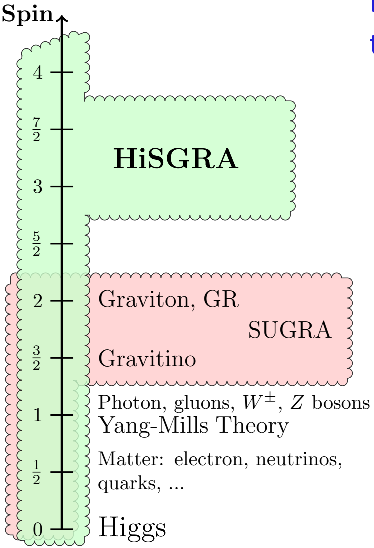
Standard context: quantum gravity problem

Standard assumption: to solve the problem without going too far from the well-established concept of particles/fields

Particles: Wigner, 1939 explained that particles = unitary irreducible representations of the space-time symmetry group (e.g. Poincare). In $4d$ one can "observe" massive or massless particles with spin $s = 0, \frac{1}{2}, 1, \ldots$ or helicity $\lambda = 0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm\mathbf{2}, \ldots$

Questions: what are the options to have interactions? Which ones incorporate gravity? Which ones are quantum consistent? Are all options accessible? ...

Spin by spin



Different spins lead to very different types of theories/physics:

- $s = 0$: Higgs
- $s = 1/2$: Matter

- $s = 1$: Yang-Mills, Lie algebras
- $s = 3/2$: SUGRA and supergeometry, graviton $\in$ spectrum
- $s = 2$: GR and Riemann Geometry, no color

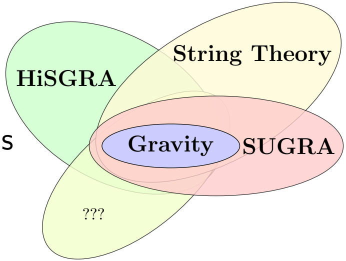
- $s > 2$: HiSGRA and String theory, ∞ states, graviton is there too!

Why massless higher spins?

- string theory
- divergences in (SU)GRA's
- Quantum Gravity via AdS/CFT

$\rightarrow$ quantization of gravity $\rightarrow$

- unbounded spin $\rightarrow \infty$ many fields
- UV $\rightarrow$ massless



HiSGRA = the smallest extensions of gravity by massless, i.e. gauge, higher spin fields. Vast gauge symmetry should render it finite.

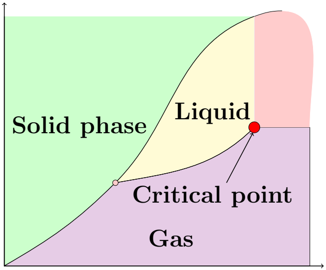
Chiral HiSGRA in brief

The smallest higher spin gravity in $4d$, $s = 0, (1), 2, (3), \ldots$; (Metsaev; Ponomarev, E.S.) can be thought of as a higher-spin extension of both SDYM and SDGR.

Admits all signs of cosmological constant $\boldsymbol{\lambda}$. It is integrable (Ponomarev) and one-loop finite (Tran, Tsulaia, E.S.) at least for $\boldsymbol{\lambda} = 0$. Relation to twistors (Tran; Herfray, Krasnov, E.S.; Tran, Adamo)

Via AdS/CFT correspondence it is dual to a subsector of (Chern-Simons) vector models ($3d$ Ising, etc.). It should imply the recently discovered $3d$ bosonization duality

Simple model of quantum gravity, which captures some physics and is related to pre-Calabi-Yau ...

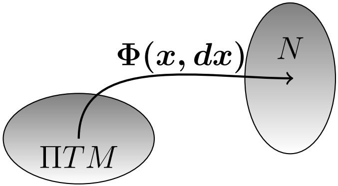


Q-manifolds, L_∞ and Formal Field Theories

Formal equations

Let us be given a Q-manifold (view it locally as an L_∞-algebra)

$$\Phi\,,\, Q^2=0$$



$$\Omega^\bullet(M)\,,\, d^2=0$$

then we can always write a sigma-model:

$$d\Phi = Q(\Phi)$$

Any PDE can be cast into such a form ... (Barnich, Grigoriev)

Other names: Free Differential Algebras (Sullivan), in physics: (van Nieuwenhuizen; Fre, D' Auria); FDA=unfolding (Vasiliev), AKSZ (AKSZ); gauged PDE (Grigoriev, Kotov); string field theory (Zwiebach)

Any PDE can be cast into such a form ...

It requires no work for Chern-Simons theory **"in any d"**

$$dA = \frac{1}{2}[A, A] = A \star A$$

where $A \equiv A_\mu \, dx^\mu$ takes values in some Lie algebra $\mathfrak{g}$ or associative.

Here, $N = (\mathfrak{g}[1], [\bullet, \bullet])$, e.g. in local coordinates A^i we have

$$Q = f^k_{ij} \, A^i A^j \frac{\partial}{\partial A^k}$$

As $\boldsymbol{L_\infty}$ we have $\boldsymbol{l_1} = 0$ and $\boldsymbol{l_2}(\bullet, \bullet) = [\bullet, \bullet]$

Any PDE can be cast into such a form ...

Flat space is still a flat connection $dA = \frac{1}{2}[A, A]$

$$de^a = \omega^{a,}{}_b \wedge e^b \qquad \text{no torsion}$$

$$\boldsymbol{d\omega^{a,b} = \omega^{a,}{}_c \wedge \omega^{c,b}} \qquad \text{Riemann vanishes}$$

vielbein $e^a \equiv e^a_\mu \, dx^\mu$; spin-connection $\omega^{a,b} \equiv \omega^{a,b}_\mu \, dx^\mu$,
so that metric tensor $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$

Minkowski space = flat Poincare algebra connection
anti-de Sitter space = flat $so(3,2) \sim sp(4)$ connection

How to add gravitational waves?

Einstein Equations

Einstein equations inside-out **Riemann**=**Weyl** instead of **Ricci** $= 0$

$$de^a = \omega^{a,}{}_b \wedge e^b \qquad \text{no torsion}$$

$$\boldsymbol{d\omega^{a,b}} = \boldsymbol{\omega^{a,}{}_c} \wedge \boldsymbol{\omega^{c,b}} + e_m \wedge e_n \boldsymbol{C^{ab,mn}} \qquad \textbf{Einstein is here! } \boldsymbol{l_3}!$$

$$d\boldsymbol{C^{ab,mn}} = Q(...) \qquad \text{Bianchi for Weyl}$$

vielbein $e^a \equiv e^a_\mu \, dx^\mu$; spin-connection $\omega^{a,b} \equiv \omega^{a,b}_\mu \, dx^\mu$;
Weyl tensor $C^{ab,cd}$ and its descendants. Riemann $=$ Weyl $+$ Ricci

As L_∞ we have $\boldsymbol{l_1} = 0$, $\boldsymbol{l_2} =$ Poincare or $so(3,2)$, $so(4,1)$ ...

$$d\omega = \boldsymbol{l_2}(\omega,\omega) + \boldsymbol{l_3(\omega,\omega,C)} + \boldsymbol{l_4}(\omega,\omega,C,C) + \ldots\,,$$
$$dC = \boldsymbol{l_2}(\omega,C) + \boldsymbol{l_3}(\omega,C,C) + \ldots\,.$$

For "typical" theories we have one-form ω and zero-form C

From L_∞ to A_∞

Why A_∞?

Recap: everything's equations of motion can be formulated as L_∞

$$d\Phi = l_2(\Phi,\Phi) + l_3(\Phi,\Phi,\Phi) + ... = Q(\Phi)$$

Often-times $\Phi = \{C, \omega\}$, i.e. $L = L_0 \oplus L_1$, note $\deg l_k = -1$.

HiSGRA are gauge theories that gauge $\mathrm{Lie}(A)$ for an associative algebra A and should be able to gauge $\mathrm{Lie}(A \otimes \mathrm{Mat}_N)$ as well, which leads to Yang-Mills gaugings and super-symmetry, if needed. $\mathrm{Lie}(A \otimes \mathrm{Mat}_N)$ knows about A.

All HiSGRA have additional structure: L_∞ **originates from an** A_∞ **via the symmetrization map**, e.g $l_2(\omega, C) = m_2(\omega, C) + m_2(C, \omega)$

Now, $m_2(\bullet,\bullet)$ defines an associative algebra A. In HiSGRA it is just $\star$-product algebra, e.g. Moyal $\star$ aka Weyl algebra $A_n \sim C[x^i, \partial_i]$

Formal integrable models from soft associative algebras

Initial data: associative algebra $\boldsymbol{A_0}$, its bimodule M, say $M \sim \boldsymbol{A_0}$:

$$m_2(a,b) = ab \quad m_2(a,u) = au \quad m_2(u,a) = -ua \quad a,b \in A^1 \quad u \in A^0$$

is an $\boldsymbol{A_\infty}$ algebra with $A^1 \sim \boldsymbol{A_0}$, $A^0 \sim M \sim \boldsymbol{A_0}$. **How to deform?**

If $\boldsymbol{A_0}$ is soft and we have a family of associative algebras $\boldsymbol{A_\nu}$

$$a \circ (b \circ c) = (a \circ b) \circ c \qquad a \circ b = ab + \sum \phi_k(a,b)\, \nu^k$$

$\phi_1 \in HH^2(\boldsymbol{A_0}, \boldsymbol{A_0})$, then we can construct all higher maps, e.g.

$$\begin{aligned} m_3(a,b,u) &= \phi_1(a,b)\, u \qquad\qquad a,b \in A^1 \quad u,v \in A^0 \\ m_4(a,b,u,v) &= \phi_2(a,b)\, u\, v + \phi_1(\phi_1(a,b),u)\, v \end{aligned}$$

The maps are nontrivial! The model is integrable!

Recap: soft associative algebras can be used to construct A_∞-algebras. Passing to L_∞ we get a formal integrable theory of type

$$d\Phi = Q(\Phi)$$

in other words, we can travel as

$$\begin{array}{ccccccc} A_0 & \to & A_\nu & \to & A_\infty & \to & L_\infty \sim Q^2 = 0 \\ m_2 & & & & m_{k>2} & & l_k \end{array}$$

Where to find soft associative algebras?

1) Deformation quantization of Poisson Manifolds. **solved!**

2) Deformation quantization of Poisson Orbifolds. **unsolved!**

3) ???

DQ of the algebra of functions on some Poisson manifold $\mathcal{M}$

$$f \star g = f \cdot g + \hbar \{f, g\} + \text{Kontsevich}$$

In the case of flat $\mathbb{R}^{2n}$ the solution is Moyal-Weyl star-product

$$(f \star g)(y) = f(y) \exp \tfrac{1}{2} \left[\overleftarrow{\partial}_A \pi^{AB} \overrightarrow{\partial}_B \right] g(y) \qquad y^A \in \mathbb{R}^{2n}$$

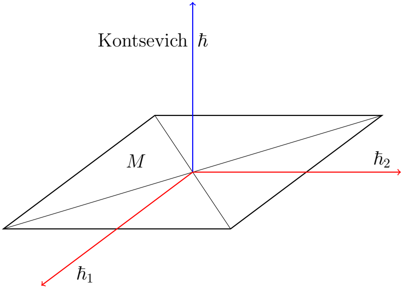
Higher spin symmetry is just a set of specific examples of DQ.

In practice, we have coadjoint orbits of, say $so(d,2)$, $so(d,1)$, i.e. Lorentz, (anti)-de Sitter algebras.

The simplest example: Moyal-Weyl $\star$-product on $\mathbb{R}^{2n}$ gives Weyl algebras $\boldsymbol{A_n} = C[q^i, p_j]$, $[p_i, q^j] = \delta_i^j$ and $sp(2n) \in \boldsymbol{A_n}$, $sp(2) \sim so(2,1)$, $sp(4) \sim so(3,2)$ — relativistic symmetries. To be gauged!

Usually, a Poisson Manifold $\mathcal{M}$ has some symmetries ...

More generally we can think of crossed product $C(M) \rtimes \Gamma$ or orbifold $C(M)/\Gamma$ for any discrete group Γ of symmetries of M



There are new directions of Deformation Quantization (Halbout, Tang).

For $\boldsymbol{A_n} \rtimes \Gamma$ explicit solution is found (Sharapov, E.S.) $\leftarrow$ (AFLS)

Open problem: Formality Conjecture for Orbifolds.

For Chiral HiSGRA we need to understand just $\mathbb{R}^2/\mathbb{Z}_2$

The Weyl algebra $\boldsymbol{A_1}$, $[q,p]=i\hbar$ is rigid, i.e. there is no deformation of $f(q,p)\star g(q,p)$ as associative algebra. But $\boldsymbol{A_1^e}$ is soft!

Suppose we need a reflection $\boldsymbol{R}f(q,p)\boldsymbol{R}=f(-q,-p)$, realized as

$$\boldsymbol{R}^2=1 \qquad \boldsymbol{R}q\boldsymbol{R}=-q \qquad \boldsymbol{R}p\boldsymbol{R}=-p$$

Algebra $A_1 \ltimes \mathbb{Z}_2$ is soft (Wigner; Yang; Mukunda; ...):

$$[q,p]=i\hbar+i\nu\boldsymbol{R}$$

Even $\boldsymbol{R}(f)=f$ lead to $gl_\lambda=U(sp_2)/(C_2-\lambda(\lambda-1))$ (Feigin) aka fuzzy-sphere. For integer $\lambda\sim\nu$ algebra gl_λ reduces to gl_n

This is thanks to $\boldsymbol{HH^2(A_1,A_1^*)=\mathbb{C}}$ and the cocycle was obtained (Feigin, Felder, Shoikhet) from Shoikhet-Tsygan-Kontsevich formality

Chiral Higher Spin Gravity
pre-Calabi-Yau
and Convex Geometry

For Chiral HiSGRA **higher-spin symmetry** is $\boldsymbol{A} = \boldsymbol{A_1^\lambda}$, $[p,q] = \boldsymbol{\lambda}$, in particular, for $\boldsymbol{\lambda} = 0$ we have commutative algebra $\mathbb{C}[q,p]$

The bimodule is $\boldsymbol{A}^*$ and $\boldsymbol{m_2}$ encode this data in $\boldsymbol{A_\infty}$

$$\text{free}: \quad \boldsymbol{d\Phi = m_2(\Phi,\Phi)} \quad \boldsymbol{\Phi} = \{\boldsymbol{A^1_\infty} \sim \omega \sim \boldsymbol{A},\ \boldsymbol{A^0_\infty} \sim C \sim \boldsymbol{A}^*\}$$

The theory gauges Weyl algebra $\boldsymbol{A}$ and we ask for interactions $\boldsymbol{m_{k>2}}$

We should also tensor this $\boldsymbol{A_\infty}$ with some associative algebra B to have the correct $4d$ interpretation. B is a free param of the model.

We have got $\boldsymbol{m_k}$ the long way and observed at the end that they form pre-Calabi-Yau algebra ...

Now, it is better to write $\mathbf{\Phi} = \{\omega_i, C^j\}$ since we have $\boldsymbol{A}$ and $\boldsymbol{A}^*$

The equations of motion are $\boldsymbol{4d}$ Poisson sigma-model

$$dC^i = \pi^{ij}(C)\,\omega_j\,, \qquad d\omega_k = \tfrac{1}{2}\partial_k\pi^{ij}(C)\,\omega_i\,\omega_j\,,$$

Master Hamiltonian $S = \frac{1}{2}\pi^{ij}(C)\,\omega_i\,\omega_j$ obeys the master equation

$$\{S,S\} = \frac{\partial S}{\partial C^i}\frac{\partial S}{\partial \omega_i} = 0 \qquad\qquad \Omega = dC^i \wedge d\omega_i$$

This is still $\boldsymbol{d\Phi = Q(\Phi)}$ for Q being the Hamiltonian vector field

$$\boldsymbol{Q} = \boldsymbol{Q^J(\Phi)}\frac{\partial}{\partial \mathbf{\Phi}^J} = \frac{\partial S}{\partial C^i}\frac{\partial}{\partial \omega_i} + \frac{\partial S}{\partial \omega_i}\frac{\partial}{\partial C^i}$$

pre-Calabi-Yau is a non-commutative Poisson structure, $[\boldsymbol{S},\boldsymbol{S}]_{nc} = 0$

Pre-Calabi-Yau/Master action

Let $y^A = \{q, p\}$ with constant $\epsilon^{AB} = -\epsilon^{BA}$. Moyal-Weyl reads

$$(f \star g)(y) = \exp[p_{01} + p_{02} + \boldsymbol{\lambda}\, p_{12}]\, f(y_1) g(y_2)\Big|_{y_i=0}$$

with $[y^A, y^B]_\star = 2\boldsymbol{\lambda}\epsilon^{AB}$ and

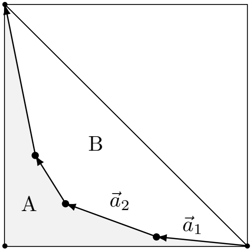
$$p_0 \equiv y \qquad\qquad p_i \equiv \partial_{y_i} \qquad\qquad p_{ij} = \epsilon_{AB}\, p_i^A p_j^B$$

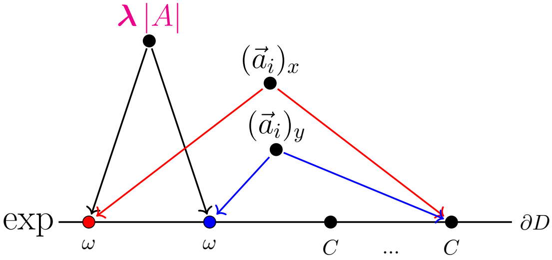
In general, we have poly-differential operators

$$S(\omega, \omega, C, ..., C) = S(p_a, p_b, p_1, ..., p_n)\, \omega(y_a)\omega(y_b)\, C(y_1)...C(y_n)\Big|_{y_\bullet=0}$$

Our $\epsilon^{AB} =$ const, so all bulk "Kontsevich" graphs simplify and the maps exponentiate

Explicit answer for all maps, e.g. $S(\omega,\omega,C,...,C)$:





(Sharapov, E.S., Van Dongen) the configuration space is of convex polygons B or swallowtails A, related to Grassmannian

$$S(p_a,p_b,p_1,...,p_n)=\int (p_{ab})^n \exp[*\boldsymbol{\lambda}\, p_{ab}+\sum *p_{ai}+\sum *p_{bi}]$$

The proof of $\boldsymbol{A_\infty}$ is by Stokes theorem: $\int_C d\Omega=\int_{\partial C}\Omega$

Collecting pieces of the puzzle

In general, we have integrable models $\boldsymbol{d\Phi = Q(\Phi)}$

$$\boldsymbol{A_0}[\hbar = \boldsymbol{\lambda}] \quad \to \quad \boldsymbol{A_\nu} \quad \to \quad \boldsymbol{A_\infty} \quad \to \quad \boldsymbol{L_\infty} \sim Q^2 = 0$$

m_2 $\qquad$ $m_{k>2}$ $\qquad$ l_k

In the pre-Calabi-Yau case of Chiral HiSGRA

$$dC^i = \pi^{ij}(C)\,\omega_j\,, \qquad\qquad d\omega_k = \tfrac{1}{2}\partial_k\pi^{ij}(C)\,\omega_i\,\omega_j\,.$$

$\boldsymbol{m_2}(a,b) =$ Moyal-Weyl, $\boldsymbol{m_3}(a,b,u) \sim \phi(a,b)\star u$ where $\phi \in HH^2(A_1, A_1^*)$ and the 2-cocycle obtained (Feigin, Felder, Shoikhet) from Shoikhet-Tsygan-Kontsevich formality

We see a small piece of a bigger formality where $\pi^{AB} =$ const, so only boundary "Kontsevich" graphs survive.

- Chiral HiSGRA is a simple model of quantum gravity that is also directly related to the physics of second order phase transitions via AdS/CFT
- Its equations of motion are given by a $4d$ Poisson sigma-model that originates from a pre-Calabi-Yau algebra, where $\boldsymbol{A} = C[y_1, y_2]$ or $\boldsymbol{A} = \boldsymbol{A_1}$ (Weyl algebra)
- $\boldsymbol{m_2} = \star$, $\boldsymbol{m_3} \sim HH^2(A, A^*)$ Shoikhet-Tsygan-Kontsevich formality/Poisson orbifolds; higher (resummed) maps = Kontsevich-like integrals over convex polygons; proof via Stokes theorem
- More: integrable models from soft associative algebras via A_∞
- We see just a tip of the iceberg ...

Thank you for your attention!

Papers

- A. Sharapov, E. Skvortsov, R. Van Dongen; **Chiral higher spin gravity and convex geometry**; arXiv:2209.01796
- A. Sharapov, E. Skvortsov, A. Sukhanov, R. Van Dongen; **More on Chiral Higher Spin Gravity and convex geometry**; arXiv:2209.15441
- E. Skvortsov, A. Sharapov; **Chiral Higher Spin Gravity in (A)dS$_4$ and secrets of Chern-Simons Matter Theories**; arXiv:2205.15293
- A. Sharapov, E. Skvortsov; **Formal Higher Spin Gravities**; arXiv:1901.01426
- A. Sharapov, E. Skvortsov; **Formal Higher-Spin Theories and Kontsevich-Shoikhet-Tsygan Formality**; arXiv:1702.08218

... backup slides ...

What Higher Spin Problem is: Field theory approach

A massless spin-s particle can be described by a rank-s tensor

$$\delta\Phi_{\mu_1\dots\mu_s} = \nabla_{\mu_1}\xi_{\mu_2\dots\mu_s} + \text{permutations}$$

which generalizes $\delta A_\mu = \partial_\mu \xi, \qquad \delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$

Fronsdal, Berends, Burgers, Van Dam, Bengtsson^2, Brink, ...

Problem: find a nonlinear completion (action, gauge symmetries)

$$S = \int (\nabla\Phi)^2 + \mathcal{O}(\Phi^3) + \dots \qquad\qquad \delta\Phi_{\dots} = \nabla_{.}\xi_{\dots} + \dots$$

and prove that it is UV-finite, hence a Quantum Gravity model

Warning: brute force does not seem to work! Bekaert, Boulanger, Leclerq; Taronna; Roiban, Tseytlin; Ponomarev; Taronna, Sleight; ...

Local HiSGRA in 2023

3d massless, conformal and partially-massless (Blencowe; Bergshoeff, Stelle; Campoleoni, Fredenhagen, Pfenninger, Theisen; Henneaux, Rey; Gaberdiel, Gopakumar; Grumiller; Grigoriev, Mkrtchyan, E.S.; Pope, Townsend; Fradkin, Linetsky; Lovrekovic; ...), $S = S_{CS}$ for a HS extension of $sl_2 \oplus sl_2$ or $so(3,2)$

$$S = \int \omega\, d\omega + \tfrac{2}{3}\omega^3$$

4d conformal (Tseytlin, Segal; Bekaert, Joung, Mourad; Adamo, Tseytlin; Basile, Grigoriev, E.S.; ...), higher spin extension of Weyl gravity, local Weyl symmetry tames non-localities

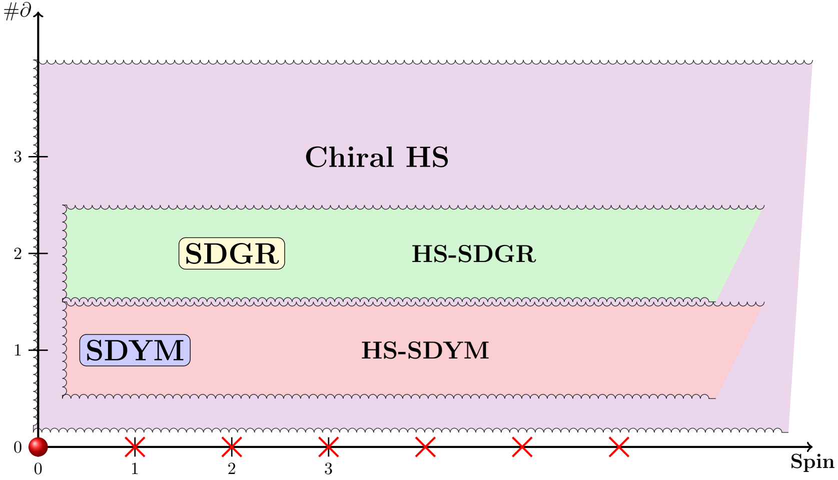
$$S = \int \sqrt{g}\,(C_{\mu\nu,\lambda\rho})^2 + ...$$

4d massless chiral (Metsaev; Ponomarev, E.S.; Ponomarev; E.S., Tran, Tsulaia, Sharapov, Van Dongen, ...). The smallest HiSGRA with propagating fields.

IKKT model for fuzzy H_4 (Steinacker, Sperling, Fredenhagen, Tran)

The theories avoid all no-go's, as close to Field Theory as possible

Russian doll of theories



Contractions (Ponomarev; Tran; Krasnov, E.S., Tran)

Higher spin symmetry: free CFT's

Let's take any free CFT, e.g. free boson or free fermion

$$\Box\phi = 0 \qquad\qquad \partial\!\!\!/\psi = 0$$

We find the stress-tensor J_{ab} and infinitely many (even spin) *higher spin conserved tensors* $J_{a_1\ldots a_s}$, aka **higher spin currents**:

$$J_s = \phi\partial...\partial\phi + ... \qquad\qquad J_s = \bar{\psi}\gamma\partial...\partial\psi + ...$$

They are quasi-primary at the unitarity bound and have $\Delta = d + s - 2$.

The associated algebra of charges is a **higher spin algebra**, not surprisingly it is given by the DQ of the orbit that corresponds to these free fields. This definition is relevant for AdS/CFT ...

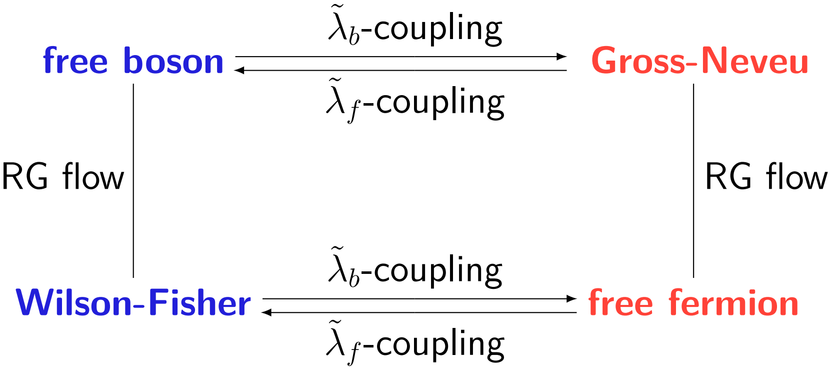
Miracle: in $3d$ $\mathfrak{hs}(\Box\phi) \sim \mathfrak{hs}(\partial\!\!\!/\psi) \sim \boldsymbol{A_2^e}$

Chern-Simons Matter theories and dualities

In AdS_4/CFT_3 one can do much better — there exists a large class of models, Chern-Simons Matter theories (extends to ABJ(M))

$$\frac{k}{4\pi}S_{CS}(A) + \text{Matter}\begin{cases}(D\phi^i)^2 & \textbf{free boson}\\ (D\phi^i)^2 + g(\phi^i\phi^i)^2 & \textbf{Wilson-Fisher (Ising)}\\ \bar{\psi}\not{D}\psi & \textbf{free fermion}\\ \bar{\psi}\not{D}\psi + g(\bar{\psi}\psi)^2 & \textbf{Gross-Neveu}\end{cases}$$

- describe physics (Ising, quantum Hall, ...)
- break parity in general (Chern-Simons)
- two parameters $\lambda = N/k$, $1/N$ (λ continuous for N large)
- exhibit remarkable dualities, e.g. ***3d* bosonization duality** (Aharony, Alday, Bissi, Giombi, Karch, Maldacena, Minwalla, Prakash, Seiberg, Tong, Witten, Yacobi, Yin, Zhiboedov, ...)



The simplest gauge-invariant operators are $J_s = \phi D ... D\phi$ or $J_s = \bar{\psi}\gamma D ... D\psi$, which are dual to higher spin fields.

Currents are slightly non-conserved $\partial \cdot J = \frac{1}{N}[JJ]$

$\gamma(J_s)$ at order $1/N$ (Giombi, Gurucharan, Kirillin, Prakash, E.S.) confirm the duality. Many other tests!